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STATIONARY FILTRATION IN A FRACTAL INHOMOGENEOUS POROUS MEDIUM

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The case of one-dimensional stationary filtration in an inhomogeneous porous medium with a fractal structure has been investigated. The fluid was assumed to be volumetrically incompressible. It has been shown that the resistance of the medium grows with increase in the fractal dimension α*, whereas the flow rate of the fluid sharply drops.*

Study of filtration in porous media with a fractal structure requires that the scope of ordinary geometric representations be exceeded and the methods of fractal theory be used [1–7]. Not uncommon are cases of disagreement of the results obtained by standard theoretical investigations with practice [3]. Apparently, the reason is that the fractal structure of a porous medium is disregarded, although in the majority of cases it has a fractal structure.

Study of filtration in a porous medium with a fractal structure has been the focus of [2, 3, 5, 6]. However, the state-of-the art of development and exploitation of oil pools and gas wells requires its further development. Therefore, study of filtration in porous media with a fractal structure and determination, on this basis, of the properties of systems is of both scientific and practical interest.

In this work, we consider the case of one-dimensional stationary filtration in an inhomogeneous medium with a fractal structure. The fluid will be assumed to be volumetrically incompressible; then the original equation will have the form [1, 2]

$$\frac{d}{dx}\left(\frac{ka_0}{\mu}\rho\frac{d^{\alpha}P}{dx^{\alpha}}\right)=0\,. \tag{1}$$

The boundary conditions are

$$P\big|_{x=0}=P_1\,,\quad P\big|_{x=l}=P_2\,. \tag{2}$$

Integrating (1), we obtain

$$\frac{d^{\alpha}P}{dx^{\alpha}}=C\,. \tag{3}$$

Integration of Eq. (3) is by the methods of integration of differential equations of fractional order according to the following formula [6]:

$$y(\xi)=b_1\frac{\xi^{\alpha-1}}{\Gamma(\alpha)}+\int\limits_0^{\xi}\frac{(\xi-t)^{\alpha-1}}{\Gamma(\alpha)}f(t,y)\,dt\,.$$

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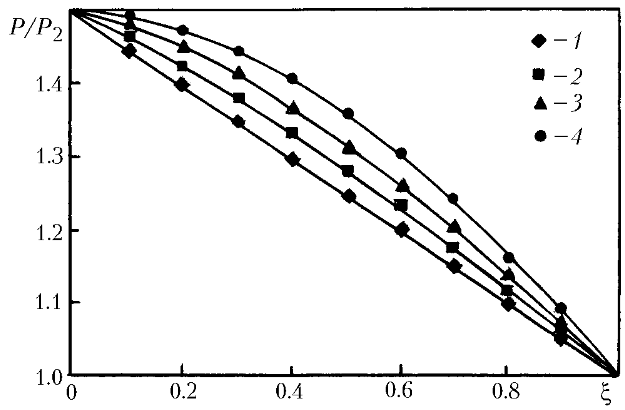

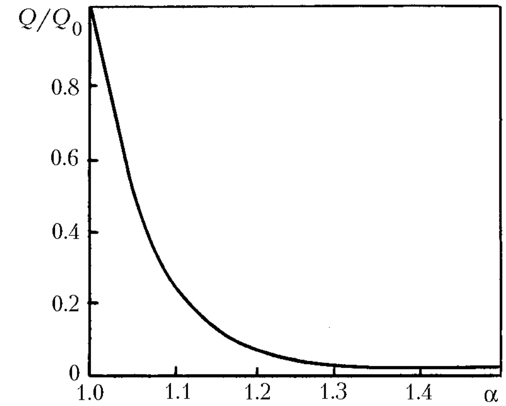

Fig. 1. Curves of distribution of the pressure P/P_2 along ξ (in dimensionless units): 1) $\alpha = 1$; 2) 1.2; 3) 1.4; 4) 1.8.

Fig. 2. Change in Q/Q_0 as a function of the fractal dimension α (in dimensionless units).

From Eq. (3) of fractional order with account for (2) we obtain [4, 6]

$$\frac{P}{P_2} = \beta + (1 - \beta)\,\xi^{\alpha}\,, \tag{4}$$

where $\xi = x/l$ and $\beta = P_1/P_2$.

The flow rate of the fluid for media with a fractal structure may be determined according to Darcy's law [1]:

$$Q = -\frac{k a_0}{\mu} f \frac{d^{\alpha} P}{d x^{\alpha}}\,. \tag{5}$$

From (5) with account for (3) and (4) we will have [6]

$$Q = \alpha \frac{k a_0}{\mu} f \frac{P_1 - \Gamma(\alpha) P_2}{l^{\alpha}}\,, \tag{6}$$

where $l^{\alpha} = (l/\delta)\delta^{\alpha}$.

Equation (6) may be reduced to the form

$$\frac{Q}{Q_0} = \frac{\alpha a_0 [\beta - \Gamma(\alpha)]}{(\beta - 1)\, l^{\alpha - 1}}\,. \tag{7}$$

The results of numerical calculations from formulas (4) and (7) for the following values of the parameters — $P_1 = 15$ MPa, $P_2 = 10$ MPa, $\beta = 1.5$, $a_0 = 1$ m$^{\alpha-1}$, $\xi = [0; 1]$, $\alpha = [1; 1.8]$, $l = 1.0$ m, and $\delta = 10^{-4}$ m — are presented in Figs. 1 and 2.

As is clear from Fig. 1, the pressure-distribution curve becomes more convex with growth in the fractal dimension of the porous medium α, i.e., the medium's resistance grows.

As follows from Fig. 2, the rate of flow of the fluid through the porous medium strongly depends on the fractal dimension α. All other things being equal, the flow rate of the fluid sharply drops with increase in α.

Thus, as the investigations carried out demonstrate, neglect of the factor of fractal structure of a porous medium may cause large errors in studying the filtration of a fluid.

NOTATION

a_0, constant coefficient; b_1 and C, constants of integration; f, cross-sectional area, m^2; h, permeability of a porous medium, m^2; l, bed length, m; P, pressure, MPa; P_1, pressure at the external boundary, MPa; P_2, bottom-hole pressure, MPa; Q, rate of flow of the fluid through an inhomogeneous porous medium with a fractal structure, m^3/sec; Q_0, rate of flow of the fluid through an inhomogeneous porous medium without a fractal structure, m^3/sec; t, variable parameter; x, coordinate; $y(\xi)$, fractional integral; α, fractal dimension of a porous medium; δ, sand-grain size, 10^{-4} m; Γ, Euler gamma function; μ, dynamic viscosity of the fluid, cP; ρ, density of the fluid, kg/m^3; ξ, dimensionless coordinate. Subscripts: 0, without a fractal structure; 1, bottom hole; 2, supply.

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